MATHEMATICS AND STATISTICS

Courses

Mathematics (MATH)

MATH 110 Linear Algebra Units: 6.00
This course is intended for students who plan to pursue a Major or Joint Honours Plan in Mathematics or Statistics. Provides a thorough introduction to linear algebra up to and including eigenvalues and eigenvectors.
Learning Hours: 264 (72 Lecture, 24 Tutorial, 168 Private Study)
Requirements: Prerequisite None. Recommended At least one 4U Mathematics course. Exclusion MATH 111/6.0*; MATH 112/3.0; MATH 212/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Understand the fundamental ideas of linear algebra, including linear systems, vector spaces, matrices, linear transformations, eigenvalues and eigenvectors, diagonalization, orthogonality, and diverse applications.
2. Give rigorous mathematical proofs of basic theorems.
3. Solve concrete problems in linear system, giving algorithmic solutions.

MATH 112 Introduction to Linear Algebra Units: 3.00
A brief introduction to matrix algebra, linear algebra, and applications. Topics include systems of linear equations, matrix algebra, determinants, the vector spaces Rn and their subspaces, bases, co-ordinates, orthogonalization, linear transformations, eigenvectors, diagonalization of symmetric matrices, quadratic forms.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite None. Recommended At least one 4U Mathematics course. Exclusion MATH 110/6.0; MATH 111/6.0*.
Course Equivalencies: MATH 110B/112 / APSC 174
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Apply the above skills to complex problems (e.g., error correcting codes, dynamical systems, games on graphs and probability).
2. Compute eigenvalues and eigenvectors and understand their utility.
3. Manipulate matrix equations and compute their determinants and inverses.
4. Solve systems of linear equations and visualize the related geometry.
5. Visualize and express algebraically the geometry of lines and planes.
6. Work with linear and affine transformations and relate them to matrices.

MATH 120 Differential and Integral Calculus Units: 6.00
This course is intended for students who plan to pursue a Major or Joint Honours Plan in Mathematics, Statistics, or Physics. A thorough discussion of calculus, including limits, continuity, differentiation, integration, multivariable differential calculus, and sequences and series.
Learning Hours: 288 (72 Lecture, 24 Tutorial, 192 Private Study)
Requirements: Prerequisite None. Recommended MHF4U and MCV4U or 4U AFIC or permission of the Department. Exclusion MATH 121/6.0; MATH 123/3.0; MATH 124/3.0; MATH 126/6.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Use the ideas in the course fluently. Indicators of fluency include: using the ideas in a new situation; using the ideas in a different order or manner than they have been presented; constructing minor extensions and variations of the ideas.
2. Write clear, clean, and well-reasoned mathematical arguments.
3. Understand the standards for such arguments.
4. Work through and solve more difficult problems, particularly those which may seem confusing at first and require time to digest and understand.
5. Demonstrate mastery of the underlying concepts of the course: limits, continuity, differentiation, integration, convergence.
6. Compute limits, derivatives, integrals, and infinite sums.

MATH 121 Differential and Integral Calculus Units: 6.00
Differentiation and integration with applications to biology, physics, chemistry, economics, and social sciences; differential equations; multivariable differential calculus.
NOTE Also offered online, consult Arts and Science Online (Learning Hours may vary).
NOTE Also offered at Bader College, UK (Learning Hours may vary).
Learning Hours: 240 (72 Lecture, 168 Private Study)
Requirements: Prerequisite None. Recommended MHF4U and MCV4U or equivalent, or 4U AFIC, or permission of the Department. Exclusion Maximum of 6.0 units from: MATH 120/6.0; MATH 121/6.0; MATH 123/3.0; MATH 124/3.0; MATH 126/6.0. Exclusion Maximum of one course from: MATH 121/6.0; MATH 130/3.0. Note This course is intended for students who wish to pursue a Major or Joint Honours Plan in a subject other than Mathematics or Statistics.
Course Equivalencies: MATH121; MATH121B;MATH122B
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Apply differential and integral calculus procedures and techniques in technical problems.
2. Extend the one-variable analysis to multi-variable functions.
3. Work with a variety of standard models and applications, using the tools of calculus to gain new understanding.
4. Communicate results in writing using appropriate mathematical format and notation.

MATH 123 Differential and Integral Calculus I Units: 3.00
Differentiation and integration of elementary functions, with applications to physical and social sciences. Topics include limits, related rates, Taylor polynomials, and introductory techniques and applications of integration.
Requirements: Prerequisite Permission of the Department. Exclusion Maximum of one course from: MATH 120/6.0; MATH 121/6.0; MATH 123/3.0; MATH 126/6.0. Exclusion Maximum of one course from: MATH 123/3.0; MATH 130/3.0. Note This course is not intended for students pursuing a MATH or STAT Plan.
Offering Faculty: Faculty of Arts and Science

MATH 124 Differential and Integral Calculus II Units: 3.00
Topics include techniques of integration; differential equations, and multivariable differential calculus.
NOTE For students who have credit for a one-term course in calculus. Topics covered are the same as those in the Winter term of MATH 121/6.0.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 123/3.0 or MATH 127/3.0 or MATH 130/3.0 or permission of the Department. Exclusion MATH 120/6.0; MATH 121/6.0; MATH 126/6.0; MATH 128/3.0.
Course Equivalencies: APSC172, MATH124
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Understand and perform the basic techniques of differential and integral calculus.
2. Apply these techniques to solve problems in the areas of biology, physics, chemistry, economics, and social sciences.
3. Solve basic problems in differential equations, multivariable differential calculus, and sequences and series.

MATH 126 Differential and Integral Calculus Units: 6.00
Differentiation and integration of the elementary functions with applications to the social sciences and economics; Taylor polynomials; multivariable differential calculus.
Learning Hours: 240 (72 Lecture, 24 Tutorial, 144 Private Study)
Requirements: Prerequisite None. Exclusion Maximum of 6.0 units from: MATH 120/6.0; MATH 121/6.0; MATH 123/3.0; MATH 124/3.0; MATH 126/6.0. Exclusion Maximum of one course from: MATH 126/6.0; MATH 130/3.0. Note This course is primarily intended for students in the BAH program. Students in the BSCH, BCMPH or BCOM program should not enrol in this course.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Apply differential and integral calculus concepts and procedures in technical problems.
2. Extend the one-variable analysis to multi-variable functions.
3. Work with a variety of standard models and applications, using the tools of calculus to gain new understanding.
4. Communicate results in writing using appropriate mathematical format and notation.

MATH 127 Differential and Integral Calculus I Units: 3.00
Differentiation and integration of the elementary functions (mainly polynomial, rational, and exponential) with applications to the social sciences and economics.
NOTE MATH 127/3.0 and MATH 128/3.0 together, are equivalent to MATH 126/6.0.
NOTE This course is primarily intended for students in the BAH program. Students in the BSCH, BCMPH, or BCOM program should not enrol in this course.
Learning Hours: 108 (36 Lecture, 72 Private Study)
Requirements: Prerequisite None. Exclusion Maximum of 6.0 units from: MATH 120/6.0; MATH 121/6.0; MATH 123/3.0; MATH 124/3.0; MATH 126/6.0; MATH 127/3.0; MATH 128/3.0. Exclusion Maximum of one course from: MATH 127/3.0; MATH 130/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Apply differential and integral calculus concepts and procedures to gain new understanding of complex systems.
2. Work with a variety of standard models from the social sciences, using the tools of calculus to solve optimization problems.
3. Communicate results in writing giving explanations that are careful, clear and elegant.

MATH 128 Differential and Integral Calculus II Units: 3.00
Multivariable functions, partial derivatives, contour diagrams, gradient, optimization, Lagrange multipliers, applications to the social sciences.
NOTE MATH 127/3.0 and MATH 128/3.0 together, are equivalent to MATH 126/6.0.
NOTE This course is primarily intended for students in the BAH program. Students in the BSCH, BCMPH, or BCOM program should not enrol in this course.
Learning Hours: 108 (36 Lecture, 72 Private Study)
Requirements: Prerequisite MATH 127/3.0 or permission of the Department. Exclusion Maximum of 6.0 units from: MATH 120/6.0; MATH 121/6.0; MATH 123/3.0; MATH 124/3.0; MATH 126/6.0; MATH 127/3.0; MATH 128/3.0. Exclusion Maximum of one course from: MATH 128/3.0; MATH 130/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Apply multivariable differential calculus concepts and procedures to gain new understanding of complex systems.
2. Work with a variety of standard models from the social sciences, using the tools of multivariable calculus to solve optimization problems.
3. Communicate results in writing giving explanations that are careful, clear and elegant.

MATH 130 Mathematics for Biochemistry and Life Sciences Units: 3.00
The course will have four topics, each approximately three weeks long. Topics include a review of functions, limits, and differentiation, antiderivatives, integration and fundamental theorem of calculus, differential equations, and probability.
Learning Hours: 120 (36 Lecture, 12 Tutorial, 72 Private Study)
Requirements: Prerequisite Registration in a BCHM or LISC Plan. Recommended 4U Calculus and Vectors (or equivalent). Exclusion MATH 121/6.0; MATH 123/3.0; MATH 126/6.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Use the ideas and methods of the differential and integral calculus to study living systems.
2. Work with the basic functions of differential calculus, in particular the exponential and logarithm functions, and solve optimization problems.
3. Work with the fundamental processes of integral calculus and differential equations.
4. Work with multivariable functions, contour diagrams and phase-plane analysis.
5. Work with probabilistic processes, and random variables in both discrete and continuous spaces, involving independent events and conditional probability.

MATH 181 Designing Sophisticated Activities for Grade 7-10 Units: 3.00
The objective of the course will be to participate in the design and construction of a new collection of mathematical problems and activities targeted for the school math curriculum at the intermediate level. The problems we work with have a rich structure that will lead us to a higher level of mathematical sophistication than is found in the classrooms today. Working in small groups, the students will be asked to construct, analyze, and present examples of their own.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (Level 3 or above and registration in a Bachelor of Education Program and [3.0 units of MATH or STAT at any level]) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Understand curriculum in terms of content, pedagogy and structure.
2. Work with the Ontario math curriculum expectations in Grades 7-10.
3. Experience teaching as guided play.
4. Work with the existing literature in terms of curriculum design.
5. Experience the power and beauty of mathematics.

MATH 210 Rings and Fields Units: 3.00
Integers, polynomials, modular arithmetic, rings, ideals, homomorphisms, quotient rings, division algorithm, greatest common divisors, Euclidean domains, unique factorization, fields, finite fields.
Learning Hours: 132 (36 Lecture, 12 Tutorial, 84 Private Study)
Requirements: Prerequisite MATH 110/6.0 or MATH 111/6.0* or (MATH 112/3.0 and MATH 212/3.0) or (MATH 112/3.0 with permission of the Department). Exclusion MATH 211/6.0*.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Perform accurate and efficient computations with integers and polynomials involving quotients, remainder, divisibility, greatest common divisors, primality, irreducibility, and factorization.
2. Define and illustrate basic concepts in ring theory using examples and counterexamples.
3. Describe and demonstrate an understanding of equivalence classes, ideals, quotient rings, ring homomorphisms, and some standard isomorphisms.
4. Recognize and explain a hierarchy of rings that includes commutative rings, unique factorization domains, principal ideal domains, Euclidean domains, and fields.
5. Write rigorous solutions to problems and clear proofs of theorems.

MATH 212 Linear Algebra II Units: 3.00
Vector spaces, direct sums, linear transformations, eigenvalues, eigenvectors, inner product spaces, self-adjoint operators, positive operators, singular-value decomposition, minimal polynomials, Jordan canonical form, the projection theorem, applications to approximation and optimization problems.
Learning Hours: 120 (36 Lecture, 12 Tutorial, 72 Private Study)
Requirements: Prerequisite MATH 111/6.0* or MATH 112/3.0 or MTHE 217/3.5. Exclusion MATH 110/6.0. Equivalency MATH 312/3.0*.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Computing eigenvalues and eigenvectors of endomorphisms.
2. Computing Jordan Normal Forms of endomorphisms.
3. Computing Taylor and Laurent expansions of complex analytic functions.
4. Proving properties of homomorphisms between vector spaces, as well as properties of eigenvalues and eigenvectors of endomorphisms.
5. Working with the properties of inner-products and Hilbert spaces.

MATH 221 Vector Calculus Units: 3.00
Double and triple integrals, including polar and spherical coordinates. Parameterized curves and line integrals. Gradient, divergence, and curl. Green's theorem. Parameterized surfaces and surface integrals. Stokes' and Gauss' Theorems.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 120 or MATH 121 or MATH 124 or MATH 126. Exclusion MATH 280. Recommended Some linear algebra.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Calculate and work with gradients, divergence and curl, and spherical and cylindrical coordinates.
2. Integrate vector functions and evaluate path integrals, surface integrals and volume integrals.
3. Perform calculations using Green's and Stokes' Theorems, and the Divergence Theorem.
4. Work with multi-variable functions and vectors in dimensions 2 and 3.

MATH 225 Ordinary Differential Equations Units: 3.00
An introduction to solving ordinary differential equations. Topics include developing and analyzing mathematical models describing physical and natural dynamical phenomena and those arising in various engineering system applications including spring-mass systems and circuits, first order differential equations, linear differential equations with constant coefficients, Laplace transforms, systems of linear equations.
NOTE Some knowledge of linear algebra is assumed.
Learning Hours: 120 (36 Lecture, 12 Tutorial, 72 Private Study)
Requirements: Prerequisite MATH 120/6.0 or MATH 121/6.0 or MATH 124/3.0 or MATH 126/6.0 or (MATH 127/3.0 and MATH 128/3.0). Exclusion MATH 231/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Model a mass-spring-damper system or RLC circuit using differential equations.
2. Model interconnected fluid reservoirs using differential equations.
3. Solve basic initial value problems.
4. Solve linear constant coefficient differential equations.
5. Use the Laplace transform to solve differential equations.

MATH 228 Complex Analysis Units: 3.00
Complex arithmetic, complex plane. Differentiation, analytic functions. Elementary functions. Contour integration, Cauchy's Theorem, and Integral Formula. Taylor and Laurent series, residues with applications to evaluation of integrals.
Learning Hours: 120 (36 Lecture, 12 Tutorial, 72 Private Study)
Requirements: Prerequisite (MATH 110/6.0 or MATH 111/6.0* or MATH 112/3.0) and (MATH 120/6.0 or MATH 121/6.0 or MATH 124/3.0). Exclusion MATH 326/3.0; PHYS 312/6.0*; PHYS 317/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Compute harmonic conjugates of harmonic functions.
2. Compute Taylor and Laurent expansions of complex analytic functions.
3. Evaluate contour integrals using residue theory.
4. Prove basic properties of complex analytic functions.
5. Solve algebraic equations involving complex numbers.
6. Understand basic properties of complex mappings.

MATH 231 Differential Equations Units: 3.00
An introduction to ordinary differential equations and their applications. Intended for students concentrating in Mathematics or Statistics.
Learning Hours: 132 (36 Lecture, 12 Tutorial, 84 Private Study)
Requirements: Prerequisite (MATH 110/6.0 or MATH 111/6.0* or MATH 212/3.0) and (MATH 120/6.0 or MATH 121/6.0 or MATH 124/3.0). Exclusion MATH 225/3.0; MATH 226/3.0*; MATH 232/3.0*.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Solve and manipulate ordinary differential equations.
2. Learn techniques to solve scalar first-order equations.
3. Learn techniques to solve linear scalar higher-order equations.
4. Learn techniques to solve linear vector first-order equations.
5. Use qualitative analysis to understand the behaviour planar systems of (not necessarily linear) ordinary differential equations.

MATH 280 Advanced Calculus Units: 3.00
Limits, continuity, C^1 and linear approximations of functions of several variables. Multiple integrals and Jacobians. Line and surface integrals. The theorems of Green, Stokes, and Gauss.
Learning Hours: 132 (36 Lecture, 12 Tutorial, 84 Private Study)
Requirements: Prerequisite (MATH 110/6.0 or MATH 111/6.0* or MATH 112/3.0) and (MATH 120/6.0 or MATH 121/6.0 or MATH 124/3.0). Exclusion MATH 221/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Computing areas of regions bounded by simple closed curves.
2. Computing path integrals.
3. Computing potential functions for conservative vector fields.
4. Determining whether or not a given force field is conservative.
5. Determining whether or not a given vector field can be the curl of another vector field.
6. Evaluating the work done by a force field along a path.
7. Using Green's theorem for computing contour integrals.

MATH 281 Introduction to Real Analysis Units: 3.00
Taylor's theorem, optimization, implicit and inverse function theorems. Elementary topology of Euclidean spaces. Sequences and series of numbers and functions. Pointwise and uniform convergence. Power series.
Learning Hours: 132 (36 Lecture, 12 Tutorial, 84 Private Study)
Requirements: Prerequisite MATH 120 or MATH 121 or MATH 124.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Determining convergence or divergence of a sequence of real numbers.
2. Determining uniform/pointwise convergence or divergence of a sequence of functions.
3. Proving properties of limits of sequences of functions.
4. Using definitions to prove relationships between types of subsets of Euclidean space.
5. Using the definition of continuity to prove properties of continuous functions.

MATH 300 Modeling Techniques in Biology Units: 3.00
Modeling will be presented in the context of biological examples drawn from ecology and evolution, including life history evolution, sexual selection, evolutionary epidemiology and medicine, and ecological interactions. Techniques will be drawn from dynamical systems, probability, optimization, and game theory with emphasis put on how to formulate and analyze models.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (MATH 120/6.0 or MATH 121/6.0 or MATH 124/3.0) and (MATH 110/6.0 or MATH 111/6.0* or MATH 112/3.0). Equivalency BIOM 300/3.0*.
Course Equivalencies: BIOM 300/3.0*, MATH 300/3.0
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Create mathematical models of simple biological interactions.
2. Use numerical and graphical methods to find approximate solutions to discrete-time and continuous-time models.
3. Analyze the long-term behaviour of nonlinear planar differential equation models.
4. Make biological inferences from the analysis of nonlinear differential equation and difference equation models.

MATH 310 Group Theory Units: 3.00
Permutation groups, matrix groups, abstract groups, subgroups, homomorphisms, cosets, quotient groups, group actions, Sylow theorems.
Learning Hours: 132 (36 Lecture, 96 Private Study)
Requirements: Prerequisite MATH 210.
Course Equivalencies: MATH310, MATH313
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Work with axioms of an abstract group, different examples of finite and infinite groups in geometric, combinatorial and algebraic settings.
2. Work with group action on sets and its orbits, the orbit-stabilizer theorem.
3. Work with homomorphism, automorphism, and isomorphism of groups, as well as all three isomorphism theorems for groups.
4. Work with Lagrange's theorem, Euler's theorem, Fermat's Little theorem, Cauchy's theorem, and their applications.
5. Work with subgroups, generators, cosets, conjugacy classes, quotient groups, as well as examples of these notions in different settings.
6. Work with Sylow theorems and their application.

MATH 311 Elementary Number Theory Units: 3.00
Congruences; Euler's theorem; continued fractions; prime numbers and their distribution; quadratic forms; Pell's equation; quadratic reciprocity; introduction to elliptic curves.

Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 210/3.0 or MATH 211/6.0*.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Gained an in introduction to advanced concepts that are taken up again in higher level courses.
2. Gained proficiency in congruence arithmetic.
3. Worked with different applications to cryptography, especially in the context of RSA encryption.

MATH 314 Representations of the Symmetric Group Units: 3.00
The symmetric group consists of all permutations of a finite set or equivalently all the bijections from the set to itself. This course explores how to map the symmetric group into a collection of invertible matrices. To handle, count, and manipulate these objects, appropriate combinatorial tools are introduced.
Learning Hours: 132 (36 Lecture, 96 Private Study)
Requirements: Prerequisite MATH 210/3.0 or MATH 211/6.0*.
Offering Faculty: Faculty of Arts and Science

MATH 326 Functions of a Complex Variable Units: 3.00
Complex numbers, analytic functions, harmonic functions, Cauchy's Theorem, Taylor and Laurent series, calculus of residues, Rouche's Theorem.
Learning Hours: 120 (36 Lecture, 12 Tutorial, 72 Private Study)
Requirements: Prerequisite MATH 281. Exclusion MATH 228; PHYS 312; PHYS 317.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Computing path integrals of analytic functions using residue theory.
2. Computing Taylor and Laurent expansions for analytic functions of a complex variable.
3. Proving results on analytic functions.
4. Proving results on roots of polynomials using complex function theory.
5. Solving algebraic equations involving complex numbers.
6. Work with precise formulation of basic definitions and results on analytic functions.
7. Work with precise use of mathematical definitions in proving results on analytic functions.

MATH 328 Real Analysis Units: 3.00
Topological notions on Euclidean spaces, continuity and differentiability of functions of several variables, uniform continuity, extreme value theorem, implicit function theorem, completeness and Banach spaces, Picard-Lindelöf theorem, applications to constrained optimization and Lagrange multipliers, and existence/uniqueness of solutions to systems of differential equations.
Learning Hours: 132 (36 Lecture, 96 Private Study)
Requirements: Prerequisite MATH 281/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Understand and distinguish various topological notions for subsets of the real line and of the n-dimensional Euclidean space, such as interior and boundary points, open/closed sets, cluster points, isolated points, nowhere dense sets, compact sets, connected sets, G-delta and F-sigma sets.
2. Understand and apply the concept of limit, continuity for functions of several real variables, and their ramifications, such as the intermediate value theorem and the extreme value theorem. Understand and apply the concept of uniform continuity and its ramification, such as the Heine-Cantor theorem. Understand and distinguish sets of continuity and their properties.
3. Understand and apply the concept of differentiability for (possibly vector-valued) functions of several real variables and its ramifications. For instance, apply the chain rule, the inverse function theorem, and the implicit function theorem in concrete examples. Understand differentiability and gradients in terms of partial derivatives.
4. Understand and apply the concept of relative extrema for functions of several variables and their relation to the Jacobian and the Hessian. Understand and apply the concept of constrained extrema and the Lagrange Multiplier Theorem in concrete examples.
5. Understand and apply the Picard-Lindelöf theorem to study the existence and uniqueness of solutions of systems of ordinary differential equations.

MATH 331 Ordinary Differential Equations II Units: 3.00
A second course on ordinary differential equations with a focus on theoretical foundations. Topics include: fundamental matrix solutions, equilibria, periodic solutions, and elementary dynamical systems.
Learning Hours: 132 (36 Lecture, 12 Tutorial, 84 Private Study)
Requirements: Prerequisite MATH 225/3.0 or MATH 231/3.0 or MTHE 225/3.5 or MTHE 235/3.5 or MTHE 237/3.5.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Understand the basic theory behind existence and uniqueness of solutions for ODEs.
2. Explore the solution space of linear systems of ODEs, and solutions of selected nonlinear ODEs.
3. Work with the notions of equilibrium and stability.
4. Work with unstable, stable and centre manifolds.
5. Work with one-parameter bifurcations.

MATH 335 Mathematics of Engineering Systems Units: 3.00
Signal Spaces (Linear Spaces, Banach and Hilbert spaces; Distributions and Schwartz space of signals). Discrete and Continuous Fourier Transforms, Laplace and Z transforms. Linear input/output systems and their stability analysis. Frequency-domain and time-domain analysis of linear time-invariant systems. Applications to modulation of communication signals, linear filter design, and digital sampling.
Learning Hours: 132 (36 Lecture, 12 Tutorial, 84 Private Study)
Requirements: Prerequisite MATH 281/3.0 and (MATH 228/3.0 or MATH 326/3.0).
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Computing the Fourier transform of a signal.
2. Solving a difference equation using the z-transform.
3. Proving results on the Fourier transform.
4. Proving results on distributions.
5. Investigating the possibility of signal representations through polynomials, Haar wavelets and harmonic signals.
6. Mathematical formulation of lowpass filtering and noise removal.
7. Mathematical analysis of signal sampling.
8. Using mathematics to develop algorithms for noise removal.

MATH 337 Stochastic Models in Operations Research Units: 3.00

Some probability distributions, simulation, Markov chains, queuing theory, dynamic programming, inventory theory.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (MATH 225/3.0 or MATH 231/3.0) and (STAT 252/3.0 or STAT 268/3.0).
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Computing optimal policies via dynamic programming in applications such as inventory control.
2. Understanding stability of queuing models.
3. Using mathematics to establish optimal decision and planning via dynamic programming.
4. Rigorously proving results in Markov chains and optimal planning/control using tools from mathematical analysis.

MATH 339 Game Theory Units: 3.00

This course highlights the usefulness of game theoretical approaches in solving problems in the natural sciences and economics. Basic ideas of game theory, including Nash equilibrium and mixed strategies; stability using approaches developed for the study of dynamical systems, including evolutionary stability and replicator dynamics; the emergence of co-operative behaviour; limitations of applying the theory to human behaviour.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (MATH 120/6.0 or MATH 121/6.0 or MATH 124/3.0) and (MATH 110/6.0 or MATH 111/6.0* or MATH 112/3.0).
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Computing expected payoffs of games.
2. Using backward induction to find solutions to games in extensive form.
3. Using iterated elimination of dominated strategies to find a solution to games in normal form.
4. Using the theorem of mixed strategy Nash equilibria to find solutions to games in normal form.
5. Finding the Nash equilibria of a game.

MATH 341 Differential Geometry Units: 3.00

Introductory geometry of curves/surfaces: directional/ covariant derivative; differential forms; Frenet formulas; congruent curves; surfaces in R3: mappings, topology, intrinsic geometry; manifolds; Gaussian/mean curvature; geodesics, exponential map; Gauss-Bonnet Theorem; conjugate points; constant curvature surfaces.
Learning Hours: 132 (36 Lecture, 96 Private Study)
Requirements: Prerequisite MATH 110 and MATH 280.
Offering Faculty: Faculty of Arts and Science

MATH 347 Introduction to Topology Units: 3.00

An introduction to point-set and algebraic topology, intended for students who want to go on to further study of geometry or analysis. Topics include topological spaces; maps; product and subspace topologies; properties of topological spaces; the fundamental group and applications.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 280/3.0 and MATH 281/3.0. Exclusion MATH 499/3.0 (Topic title: Introduction to Topology - Fall 2019, Winter 2024); MATH 499/3.0 (Topic title: Introduction to Topology and Metric Spaces - Fall 2022).
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Use the ideas in the course fluently. Indicators of fluency include: being able to use the ideas in a new situation; being able to use the ideas in a different order or manner than they have been presented; being able to construct minor extensions or use minor variations of the ideas.
2. Write clear, clean, and well-reasoned mathematical arguments.
3. Understand the standards for such arguments.
4. Master the ideas of topological spaces, subspaces, product and quotient spaces, continuity, compactness, separation and countability conditions, metric spaces and metrizability, the fundamental group and some applications.

MATH 381 Mathematics with a Historical Perspective Units: 3.00

A historical perspective on mathematical ideas focusing on a selection of important and accessible theorems. A project is required.
Learning Hours: 120 (36 Lecture, 12 Group Learning, 72 Private Study)
Requirements: Prerequisite (MATH 110/6.0 or MATH 111/6.0* or MATH 212/3.0) and (MATH 120/6.0 or MATH 121/6.0 or MATH 126/6.0).
Offering Faculty: Faculty of Arts and Science

MATH 382 Mathematical Explorations Units: 3.00
Elementary mathematical material will be used to explore different ways of discovering results and mastering concepts. Topics will come from number theory, geometry, analysis, probability theory, and linear algebra. Much class time will be used for problem solving and presentations by students.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (MATH 221/3.0 or MATH 225/3.0 or MATH 231/3.0 or MATH 280/3.0 or MATH 232/3.0*) and (MATH 210/3.0 or MATH 211/6.0*).
Offering Faculty: Faculty of Arts and Science

MATH 384 Mathematical Theory of Interest Units: 3.00
Interest accumulation factors, annuities, amortization, sinking funds, bonds, yield rates, capital budgeting, contingent payments. Students will work mostly on their own; there will be a total of six survey lectures and six tests throughout the term, plus opportunity for individual help.
Learning Hours: 120 (24 Lecture, 96 Private Study)
Requirements: Prerequisite Level 3 or above and (MATH 120/6.0 or MATH 121/6.0 or MATH 124/3.0 or MATH 126/6.0).
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Define and recognize the definitions of annuity-immediate, annuity due, perpetuity, m-thly payable, continuous annuity, level payment annuity, arithmetic increasing/decreasing annuity, geometric increasing/decreasing annuity, term of annuity. Given sufficient information of immediate or due, present value, future value, current value, interest rate, payment amount, and term of annuity, student is able to calculate any remaining item.
2. Understand different type of interest rates: effective rate, nominal rate, discount rate, simple rate and simple discount, real and inflation rates, yield rate, and be able to set up the equation of values and perform calculations relating to present value, current value, and accumulated value.
3. Understand key concepts of bonds: term of bond, bond price, book value, redemption value, face value, yield rate, coupon, coupon rate, term of bond, callable bond, amortization of bond. Given sufficient information of bond, be able to calculate the remaining item(s).
4. Understand key concepts of cash flows: cash-in, cash-out, net cash flow, yield rates, net present value, and internal rate of return, measure of duration and convexity, cashflow matching and immunization. Be able to perform related calculations.
5. Understand key concepts of loans: amortization, term of loan, outstanding balance, principal repayment, interest amount/payment, payment period, refinancing. Given sufficient information of loans, be able to calculate any remaining item(s).

MATH 385 Life Contingencies Units: 3.00
Measurement of mortality, life annuities, life insurance, premiums, reserves, cash values, population theory, multi-life functions, multiple-decrement functions. The classroom meetings will be primarily problem-solving sessions, based on assigned readings and problems.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite ([MATH 120/6.0 or MATH 121/6.0 or MATH 124/3.0 or MATH 126/6.0] and [STAT 268/3.0 or STAT 252/3.0]) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Understand the mathematical theory behind life contingencies.
2. Work with survival models, life insurance, life annuities, benefit premiums, benefit reserves, multiple life functions, multiple decrement models and insurance models including expenses.
3. Gain a significant start on preparations for the Society of Actuaries MLC examination.

MATH 386 Our Number System - an Advanced Perspective Units: 3.00
Integers and rationals from the natural numbers; completing the rationals to the reals; consequences of completeness for sequences and calculus; extensions beyond rational numbers, real numbers, and complex numbers.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 281.
Offering Faculty: Faculty of Arts and Science

MATH 387 Elementary Geometry - an Advanced Perspective Units: 3.00
In-depth follow-up to high school geometry: striking new results/connections; analysis/proof of new/familiar results from various perspectives; extensions (projective geometry, e.g.); relation of classical unsolvable constructions to modern algebra; models/technology for geometric exploration.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (Level 3 or above and [MATH 221 or MATH 280 or MATH 281]) or permission of the Department.
Offering Faculty: Faculty of Arts and Science

MATH 401 Graph Theory Units: 3.00
An introduction to graph theory, one of the central disciplines of discrete mathematics. Topics include graphs, subgraphs, trees, connectivity, Euler tours, Hamiltonian cycles, matchings, independent sets, cliques, colourings, and planarity.
NOTE Given jointly with MATH 801.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 210/3.0 or MATH 211/6.0*. Recommended Experience with abstract mathematics and mathematical proof, and a good foundation in linear algebra.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Experience the development of other topics such as Ramsey theory, spectral methods, or random graphs.
2. Use an inquiry-based approach to explore bipartite graphs, trees and connectivity, Euler and Hamiltonian paths, graph matchings and colourings, and planar graphs.
3. Work with the fundamental concepts of graph theory (cycles, regular graphs, matrix representations, isomorphisms, etc.).

MATH 402 Enumerative Combinatorics Units: 3.00
Enumerative combinatorics is concerned with counting the number of elements of a finite set. The techniques covered include inclusion-exclusion, bijective proofs, double-counting arguments, recurrence relations, and generating functions.
NOTE Given jointly with MATH 802.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 210/3.0 or MATH 211/6.0*. Recommended Experience with abstract mathematics and mathematical proof, and a good foundation in linear algebra.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Understand the role of generating functions in combinatorial analysis.
2. With the study of examples and proofs, interact with a variety of concepts and techniques from enumerative combinatorics.
3. Work with counting techniques, permutations, partitions, cardinality, and Fibonacci and Catalan numbers.

MATH 406 Introduction to Coding Theory Units: 3.00
Construction and properties of finite fields. Polynomials, vector spaces, block codes over finite fields. Hamming distance and other code parameters. Bounds relating code parameters. Cyclic codes and their structure as ideals. Weight distribution. Special codes and their relation to designs and projective planes. Decoding algorithms.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 210.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Do matrix manipulations for linear codes and to compute decoding errors.
2. Rigorously prove results on error correction and detection.
3. Understand the structure of finite fields and to do computations in these fields.
4. Understand various methods for encoding and decoding messages for the purpose of error-correction and to perform the necessary computations.

MATH 413 Introduction to Algebraic Geometry Units: 3.00
An introduction to the study of systems of polynomial equations in one or many variables. Topics covered include the Hilbert basis theorem, the Nullstellenstaz, the dictionary between ideals and affine varieties, and projective geometry.
Learning Hours: 132 (36 Lecture, 96 Private Study)
Requirements: Prerequisite MATH 210.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Create and present rigorous solutions to problems and coherent proofs of theorems.
2. Define and illustrate the correspondence between ideals and varieties by translating between algebraic and geometric statements.
3. Describe and demonstrate a basic understanding of projective geometry.
4. Execute accurate and efficient calculations with ideals in a multivariate polynomial ring involving Gröbner bases, membership, intersections, and quotients.
5. Explain and use elimination theory to solve systems of polynomial equations.

MATH 414 Introduction to Galois Theory Units: 3.00
An introduction to Galois Theory and some of its applications.

Learning Hours: 132 (36 Lecture, 96 Private Study)
Requirements: Prerequisite MATH 310.
Offering Faculty: Faculty of Arts and Science

MATH 418 Number Theory and Cryptography Units: 3.00
Time estimates for arithmetic and elementary number theory algorithms (division algorithm, Euclidean algorithm, congruences), modular arithmetic, finite fields, quadratic residues. Simple cryptographic systems; public key, RSA. Primality and factoring: pseudoprimes, Pollard's rho-method, index calculus. Elliptic curve cryptography.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 210/3.0 or (MATH 211/6.0* with permission of the Department).
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Design a cryptographic system with given real-world parameters.
2. Perform analysis of cryptographic protocols.
3. Perform computations and constructions in finite fields.
4. Perform probability analysis of primality tests.
5. Perform reasoning on elliptic curves.
6. Rigorously prove results in number theory.
7. Test various designs to determine which performs the best.
8. Understand the algebraic structure of a group.

MATH 421 Fourier Analysis Units: 3.00
An exploration of the modern theory of Fourier series: Abel and Cesàro summability; Dirichlet's and Fejér's kernels; term by term differentiation and integration; infinite products; Bernoulli numbers; the Fourier transform; the inversion theorem; convolution of functions; the Plancherel theorem; and the Poisson summation theorem.
Learning Hours: 132 (36 Lecture, 96 Private Study)
Requirements: Prerequisite MATH 281 or permission of the Department.
Offering Faculty: Faculty of Arts and Science

MATH 427 Introduction to Deterministic Dynamical Systems Units: 3.00
Topics include: global properties of flows and diffeomorphisms, Invariant sets and dynamics, Bifurcations of fixed and periodic points; stability and chaos. Examples will be selected by the instructor. Given jointly with MATH 827.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite ([MATH 225/3.0 or MATH 231/3.0] and MATH 328/3.0) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Explore a variety of different types of dynamical systems: flows and semiflows, differential and difference equations, vector fields, linear versions of the above.
2. Investigate various classes of low-dimensional dynamical systems, and consider in detail what behaviours can be exhibited by these.
3. Study recurrent behaviour, indecomposability, invariance and stability.
4. Understand what a dynamical system is.

MATH 429 Functional Analysis and Quantum Mechanics Units: 3.00
A generalization of linear algebra and calculus to infinite dimensional spaces. Now questions about continuity and completeness become crucial, and algebraic, topological, and analytical arguments need to be combined. We focus mainly on Hilbert spaces and the need for Functional Analysis will be motivated by its application to Quantum Mechanics.
Learning Hours: 132 (36 Lecture, 12 Group Learning, 84 Private Study)
Requirements: Prerequisite ([MATH 110/6.0 or MATH 111/6.0* or MATH 112/3.0] and MATH 281/3.0) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Have experience with extensions to the mathematics of Quantum Mechanics, the Schrödinger equations, and the harmonic oscillator.
2. Have foundational experience with metric spaces, Banach spaces, inner product spaces, Hilbert spaces, operators and their spectrum.
3. Work with extending the concepts of Linear Algebra and Calculus to the infinite dimensional setting.
4. Work with problems from Mathematical Physics and in particular with the mathematical foundations of Quantum Mechanics.

MATH 430 Control Theory Units: 3.00
This course covers core topics in both classical and modern control theory. Overview of classical control theory using frequency methods. Linear and nonlinear controlled differential systems and their solutions. Stabilization and stability methods via Lyapunov analysis or linearization. Controllability, observability, minimal realizations, feedback stabilization, observer design. Optimal control theory, the linear quadratic regulator, dynamic programming.
NOTE Given jointly with MATH 830.
Learning Hours: 120 (36 Lecture, 12 Tutorial, 72 Private Study)
Requirements: Prerequisite (MATH 212/3.0 and [MATH 225/3.0 or MATH 231/3.0] and [MATH 326/3.0 or MATH 328/3.0]) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Linearize a nonlinear system around a given trajectory.
2. Construct LTI realizations of given transfer functions.
3. Determine the controllability/observability properties of a linear time varying system.
4. Rigorously use concepts from linear algebra/differential equations in proving results on linear control systems.
5. Use tools from optimal control theory in order to solve optimization problems.
6. Use tools from linear algebra in order to construct stabilizing controllers.
7. Study linear approximations to nonlinear systems and the limits of validity of linearization.
8. Study finite dimensional linear approximations to infinite dimensional linear systems and the limits of validity of that approximation.

MATH 433 Continuum Mechanics with Applications Units: 3.00
Continuum mechanics lays the foundations for the study of the mechanical behavior of materials. After a review of vector and tensor analysis, the kinematics of continua are introduced. Conservation of mass, balance of momenta and energy are presented with the constitutive models. Applications are given in elasticity theory and fluid dynamics.
NOTE This is the MATH version of MTHE 433 in FEAS.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite ([MATH 225/3.0 or MATH 231/3.0] and MATH 280/3.0) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Compute tractions, principal stresses, and principal axes of stress of a body.
2. Derive and analyze the differential equations governing the motion of a body.
3. Develop engineering solutions for real-world problems concerned with stress of materials, deformation of elastic bodies and motion of fluids.
4. Understand definitions and techniques of tensor algebra and tensor calculus.
5. Use the methods of tensor algebra and calculus for the description of the deformation of a body.
6. Use the methods of tensor algebra and calculus to describe the motion of a body.

MATH 434 Optimization Theory with Applications to Machine Learning Units: 3.00
Theory of convex sets and functions; separation theorems; primal-duel properties; geometric treatment of optimization problems; algorithmic procedures for solving constrained optimization programs; engineering and economic applications.
Learning Hours: 132 (36 Lecture, 96 Private Study)
Requirements: Prerequisite (MATH 110/6.0 or MATH 111/6.0* or MATH 212/3.0) and MATH 281/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Computing necessary conditions for optimality.
2. Solving constrained optimization problems.
3. Understanding the mathematical properties of convex sets and convex functions.
4. Rigorously using separation theorems for solving optimization problems.
5. Using numerical methods in the study of optimization problems.
6. Solving resource allocation problems using duality theory.

MATH 436 Partial Differential Equations Units: 3.00
Well-posedness and representation formulae for solutions to the transport equation, Laplace equation, heat equation, and wave equation. Fundamental solutions. Properties of harmonic functions. Green's function. Mean value formulae. Energy methods. Maximum principles. Method of characteristics for quasilinear equations. Burgers' equation. Shocks formation and entropy condition. Applications to fluid dynamics, elasticity problems and/or optimization problems.
Learning Hours: 132 (36 Lecture, 96 Private Study)
Requirements: Prerequisite ([MATH 225/3.0 or MATH 231/3.0] and MATH 280/3.0) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Solve and analyze the partial differential equations modeling transport phenomena.
2. Solve and analyze the partial differential equations modeling diffusion phenomena.
3. Solve and analyze the initial-boundary value problems involving Laplace equation and Poisson equation.
4. Solve and analyze the partial differential equations modeling waves and vibrations.
5. Use the method of characteristics to solve first-order quasilinear equations.
6. Apply analytical tools to solve nonlinear partial differential equations.

MATH 439 Lagrangian Mechanics, Dynamics, and Control Units: 3.00
Geometric modeling, including configuration space, tangent bundle, kinetic energy, inertia, and force. Euler-Lagrange equations using affine connections. The last part of the course develops one of the following three applications: mechanical systems with nonholonomic constraints; control theory for mechanical systems; equilibria and stability.
Learning Hours: 132 (36 Lecture, 12 Tutorial, 84 Private Study)
Requirements: Prerequisite ([MATH 225/3.0 or MATH 231/3.0] and [MATH 280/3.0 or MATH 281/3.0]) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Model physical systems using methods of geometric mechanics.
2. Translate physical concepts to differential geometric concepts.
3. Understand the definitions and constructions from differential geometry.
4. Use the methods of differential geometry.

MATH 455 Stochastic Processes and Applications Units: 3.00
Markov chains, birth and death processes, random walk problems, elementary renewal theory, Markov processes, Brownian motion and Poisson processes, queuing theory, branching processes.
NOTE This course is also listed/offered as STAT 455/3.0.
Learning Hours: 120 (36 Lecture, 12 Tutorial, 72 Private Study)
Requirements: Prerequisite STAT 353/3.0. Exclusion STAT 455/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Computing an expectation using conditioning.
2. Computing an expectation using Markov Chain Monte Carlo.
3. Converting a process description into a Markov chain model.
4. Identifying the stationary distribution of Markov chains.
5. Proving results about Markov chains.
6. Understanding the mathematical structure of a Markov chain.

MATH 472 Optimization and Control of Stochastic Systems Units: 3.00
Optimization, control, and stabilization of dynamical systems under probabilistic uncertainty with applications in engineering systems and applied mathematics. Topics include controlled and control-free Markov chains, stochastic stability, martingale methods for stability, stochastic learning, dynamic programming, optimal control for finite and infinite horizons, average cost problems, partially observed models, non-linear and Kalman filtering, linear programming and numerical methods, reinforcement learning and stochastic approximation methods, decentralized and continuous time stochastic control.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 281/3.0 and (STAT 252/3.0 or STAT 268/3.0).
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Understand stochastic stability notions for controlled and control-free Markov chains, including recurrence, positive Harris recurrence, and transience and stationarity.
2. Establish structural and existence results on optimal control policies through dynamic programming and properties of conditional expectation.
3. Understand the theory of martingales and their use in optimal stochastic control.
4. Rigorously use tools from stochastic analysis and stochastic control theory.
5. Compute optimal policies and costs for stochastic control problems via various analytical, numerical, and computational methods (including machine learning and reinforcement learning methods).
6. Design controllers leading to various forms of stability for a controlled stochastic system.

MATH 474 Information Theory Units: 3.00
Topics include: information measures, entropy, mutual information, modeling of information sources, lossless data compression, block encoding, variable-length encoding, Kraft inequality, fundamentals of channel coding, channel capacity, rate-distortion theory, lossy data compression, rate-distortion theorem. Given jointly with MATH 874.
Learning Hours: 140 (36 Lecture, 104 Private Study)
Requirements: Prerequisite STAT 268/3.0 or STAT 252/3.0. Recommended STAT 353/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Computing Shannon's information measures (entropy, Kullback-Leibler distance and mutual information).
2. Computing the capacity of communication channels.
3. Reasoning about the properties of Shannon's information measures (entropy, Kullback-Leibler distance and mutual information).
4. Using mathematical tools to infer properties of coding and communication systems.
5. Working with probabilistic modeling of communication systems for source and channel coding purposes.
6. Using tools from probability theory to analyze communication systems.
7. Working with metric assessment of data compression code designs.

MATH 477 Data Compression and Source Coding: Theory and Algorithms Units: 3.00
Topics include: arithmetic coding, universal lossless coding, Lempel-Ziv and related dictionary based methods, rate distortion theory, scalar and vector quantization, predictive and transform coding, applications to speech and image coding.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 474/3.0. Recommended STAT 353/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Compute distortion in source quantization.
2. Prove rigorously the optimality of lossless and lossy source codes.
3. Work with mathematical formulation of source coding.
4. Use probabilistic tools to understand the effect of data compression on random sources.

MATH 487 Stochastic Calculus with Applications to Mathematical Finance Units: 3.00
This course provides a rigorous introduction to the Itô Stochastic Calculus, with applications to Mathematical Finance. Topics include: measure-theoretic probability, discrete and continuous-time martingales and stopping times, Doob's Optional Sampling Theorem and Maximal Inequalities, martingale convergence theorems, Brownian motion, predictable processes, the Itô stochastic integral, local martingales and semimartingales, the quadratic variation process of a local martingale, the Itô formula, applications to mathematical finance (the Black-Scholes equation for option pricing, the Greeks).
Learning Hours: 132 (36 Lecture, 96 Private Study)
Requirements: Prerequisite (MATH 110/6.0 and MATH 281/3.0 and STAT 268/3.0) or permission of the Department. Exclusion MATH 437/3.0*.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Have a rigorous understanding of the key elements of the theory of martingales (sub- and super-).
2. Rigorously use key results from martingale theory in order to establish properties of stochastic processes, and, in particular, of Brownian motion.
3. Rigorously construct the Itô stochastic integral of a predictable process with respect to a right-continuous square-integrable martingale.
4. Rigorously apply the Itô stochastic calculus to solve problems in stochastic analysis.
5. Have a rigorous understanding of stochastic differential equations.
6. Rigorously apply tools from stochastic analysis to solve problems in mathematical finance.

MATH 497 Topics in Mathematics IV Units: 3.00
An important topic in mathematics not covered in any other courses.
Requirements: Note The prerequisite can vary depending on specific course content, please consult the Instructor or visit the Department of Mathematics and Statistics webpage for more information.
Offering Faculty: Faculty of Arts and Science

MATH 499 Topics in Mathematics Units: 3.00
Important topics in mathematics not covered in any other courses.
NOTE This course is repeatable for credit under different topic titles.
Requirements: Prerequisite Permission of the Department.
Offering Faculty: Faculty of Arts and Science

MATH 594 Independent Study Units: 3.00
Exceptionally qualified students entering their third- or fourth-year may take a program of independent study provided it has been approved by the Department or Departments principally involved. The Department may approve an independent study program without permitting it to be counted toward a concentration in that Department. It is, consequently, the responsibility of students taking such programs to ensure that the concentration requirements for their degree will be met.
NOTE Requests for such a program must be received one month before the start of the first term in which the student intends to undertake the program.
Requirements: Prerequisite Permission of the Department or Departments principally involved.
Offering Faculty: Faculty of Arts and Science

MATH 595 Independent Study Units: 6.00
Exceptionally qualified students entering their third- or fourth-year may take a program of independent study provided it has been approved by the Department or Departments principally involved. The Department may approve an independent study program without permitting it to be counted toward a concentration in that Department. It is, consequently, the responsibility of students taking such programs to ensure that the concentration requirements for their degree will be met.
NOTE Requests for such a program must be received one month before the start of the first term in which the student intends to undertake the program.
Requirements: Prerequisite Permission of the Department or Departments principally involved.
Offering Faculty: Faculty of Arts and Science

Statistics (STAT)

STAT 161 Introduction to Data Science Units: 3.00
This course introduces critical concepts, tools, techniques and skills in statistical inference/learning, machine learning, and computer programming, through hands-on analysis of real-world datasets from diverse fields in science and social science. It offers three perspectives (inferential thinking, computational thinking and real-world relevance) on the foundations of Data Science and develops a data-oriented mindset.
Learning Hours: 132 (36 Lecture, 12 Laboratory, 84 Private Study)
Requirements: Prerequisite None. Recommended An Ontario 4U mathematics course or equivalent. Exclusion Maximum of one course from: BIOL 243/3.0; CHEE 209/3.5; CISC 171/3.0; COMM 162/3.0; ECON 250/3.0; GPHY 247/3.0; HSCI 190/3.0; KNPE 251/3.0; NURS 323/3.0; POLS 285/3.0; POLS 385/3.0*; PSYC 202/3.0; SOCY 211/3.0; STAM 200/3.0; STAT 161/3.0; STAT 263/3.0. Exclusion Maximum of one course from: PATH 111/3.0; STAT 161/3.0. One-Way Exclusion May not be taken after STAT 269/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Work with critical concepts, tools, techniques, and skills in computer programming, statistical inference/learning and machine learning.
2. Use visualization to understand data.
3. Work with the computational tools and practices for summary, analysis, and visualization of data.
4. Analyze real data sets and communicate their results.
5. Have a basic understanding of the implications and tools of data collection.

STAT 252 Introductory Applied Probability Units: 3.00
Basics of probability. Counting principle, binomial expansion. Conditional probability and Bayes' Theorem. Random variables, mean and variance. Bernoulli, binomial, geometric, hypergeometric and exponential distributions. Poisson approximation. Distribution, frequency and density functions. Normal distribution and central limit theorem.
NOTE STAT 252 is a new course for STAT Minors and Joint Honours.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite MATH 120/6.0 or MATH 121/6.0 or MATH 126/6.0 or MATH 124/3.0. Exclusion STAT 268/3.0; STAT 351/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Have experience working with Bernoulli and binomial distributions, negative binomial and geometric distributions, hypergeometric distribution, Poisson distribution, the normal distribution, the central limit theorem.
2. Have experience working with discrete random variable, continuous random variables, expectation, moments, Chebyshev's theorem, moment-generating-functions.
3. Have experience working with sample spaces, events, probability of an event; review of set notation, counting rules and combinatorial methods, rules of probability conditional probability; independent events, Bayes' theorem.
4. Understand the fundamental concepts in probability with an emphasis on inquiry-based problem solving.

STAT 263 Introduction to Statistics Units: 3.00
A basic course in statistical methods with the necessary probability included. Topics include probability models, random variables, distributions, estimation, hypothesis testing, elementary nonparametric methods.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite None. Recommended An Ontario 4U mathematics course or equivalent. Exclusion Maximum of one course from: BIOL 243/3.0; CHEE 209/3.5; CISC 171/3.0; COMM 162/3.0; ECON 250/3.0; GPHY 247/3.0; HSCI 190/3.0; KNPE 251/3.0; NURS 323/3.0; POLS 285/3.0; POLS 385/3.0*; PSYC 202/3.0; SOCY 211/3.0; STAM 200/3.0; STAT 161/3.0; STAT 263/3.0. One-Way Exclusion May not be taken with or after STAT 269/3.0. Note This course is not open to Commerce students.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Calculate and display descriptive statistics for data presented in several formats.
2. Understand and apply basic concepts of probability.
3. Understand the law of large numbers and the central limit theorem.
4. Understand concepts of discrete and continuous random variables and probability distributions.
5. Understand how classical distributions such as the Normal Distribution, t, F, and $\chi 2$ are used in interval estimation and hypothesis testing.
6. Discern which of twenty-two basic hypothesis tests is appropriate in a given scenario.

STAT 268 Statistics and Probability I Units: 3.00
Basic ideas of probability theory such as random experiments, probabilities, random variables, expected values, independent events, joint distributions, conditional expectations, moment generating functions. Main results of probability theory including Chebyshev's inequality, law of large numbers, central limit theorem. Introduction to statistical computing.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (MATH 120 or MATH 121 or MATH 122 or MATH 124). Corequisite (MATH 221 or MATH 280). Exclusion STAT 252; STAT 351.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Calculate expected values.
2. Calculate probabilities of interest.
3. Understand and apply basic concepts of probability.
4. Understand and apply concepts of joint, marginal and conditional distributions.
5. Understand concepts of random variables and probability distributions.

STAT 269 Statistics and Probability II Units: 3.00
Basic techniques of statistical estimation such as best unbiased estimates, moment estimates, maximum likelihood. Bayesian methods. Hypotheses testing. Classical distributions such as the t-distribution, F-distribution, beta distribution. These methods will be illustrated by simple linear regression. Statistical computing.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (MATH 221 or MATH 280) and (STAT 252 or STAT 268 or STAT 351) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Be able to find the distribution of functions of random variables, and understand how classical distributions such as t-distribution, F-distribution and $\chi 2$ distribution are defined.
2. Understand basic statistical estimation procedures, including maximum likelihood estimation and method of moments.
3. Understand the concept of hypothesis testing and be able to apply appropriate statistical tests for comparing means, proportions and variances.
4. Understand the concept of interval estimation and be able to find the confidence intervals of means, proportions and variances.
5. Understand the law of large numbers and the central limit theorem and how they are applied in the development of statistical theory.

STAT 353 Probability II Units: 3.00
Intermediate probability theory as a basis for further study in mathematical statistics and stochastic processes; probability measures, expectations; modes of convergence of sequences of random variables; conditional expectations; independent systems of random variables; Gaussian systems; characteristic functions; law of large numbers; central limit theory; some notions of dependence.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (MATH 110/6.0 or MATH 111/6.0* or MATH 112/3.0) and (STAT 252/3.0 or STAT 268/3.0) and MATH 281/3.0.
Offering Faculty: Faculty of Arts and Science

STAT 361 Applied Methods in Statistics I Units: 3.00
A detailed study of simple and multiple linear regression, residuals and model adequacy. The least squares solution for the general linear regression model. Analysis of variance for regression and simple designed experiments; analysis of categorical data.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (MATH 110/6.0 or MATH 111/6.0* or MATH 112/3.0) and (STAT 252/3.0 or STAT 268/3.0 or STAT 351/3.0*) and (STAT 263/3.0 or STAT 269/3.0) or permission of the Department. Exclusion ECON 351/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Applying analysis of variance to understand the sources of uncertainty; applying least square criterion to build fitted models.
2. Conducting data analysis using linear regression models and reporting the analysis results.
3. Creating scatter plots, boxplots, pairwise plots to explore the data; analyzing data using linear regression models; obtaining least square estimates; comparing two different linear regression models; choosing significant variables; performing regression diagnostics for assessing the adequacy of models; drawing conclusions based on the analysis results.
4. Evaluating the appropriateness of using linear regression models in real applications.
5. Using built-in functions in R software to perform linear regression analysis.
6. Writing independent functions and codes in R to conduct linear regression analysis.

STAT 362 R for Data Science Units: 3.00
Introduction to R, data creation and manipulation, data import and export, scripts and functions, control flow, debugging and profiling, data visualization, statistical inference, Monte Carlo methods, decision trees, support vector machines, neural network, numerical methods.
Learning Hours: 118 (36 Lecture, 12 Group Learning, 70 Private Study)
Requirements: Prerequisite (STAT 252/3.0 or STAT 263/3.0 or STAT 268/3.0 or STAT 351/3.0*) and (MATH 110/6.0 or MATH 111/6.0* or MATH 120/6.0 or MATH 121/6.0 or MATH 124/3.0 or MATH 126/6.0 or [MATH 112/3.0 and MATH 212/3.0]) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Apply appropriate methods for statistical analysis and interpret the output.
2. Apply common machine learning algorithms with real-world applications.
3. Import and tidy data for further analysis.
4. Visualize data and perform exploratory data analysis.
5. Understand the basics of numerical and Monte Carlo methods.
6. Understand the fundamental concepts in programming and R.

STAT 455 Stochastic Processes and Applications Units: 3.00
Markov chains, birth and death processes, random walk problems, elementary renewal theory, Markov processes, Brownian motion and Poisson processes, queuing theory, branching processes.
NOTE This course is also listed/offered as MATH 455/3.0.
Learning Hours: 120 (36 Lecture, 12 Tutorial, 72 Private Study)
Requirements: Prerequisite STAT 353/3.0. Exclusion MATH 455/3.0.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Computing an expectation using conditioning.
2. Computing an expectation using Markov Chain Monte Carlo.
3. Converting a process description into a Markov chain model.
4. Identifying the stationary distribution of Markov chains.
5. Proving results about Markov chains.
6. Understanding the mathematical structure of a Markov chain.

STAT 456 Bayesian Analysis Units: 3.00
An introduction to Bayesian analysis and decision theory; elements of decision theory; Bayesian point estimation, set estimation, and hypothesis testing; special priors; computations for Bayesian analysis. Given Jointly with STAT 856.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite STAT 463 or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Demonstrate proficiency in finding the Fisher information contained in the data about unknown parameters.
2. Find Bayesian estimators for different functions of unknown parameters, under various loss functions.
3. Find the best unbiased estimators in the Hardy-Weinberg genetic equilibrium model.
4. Identify least informative prior distributions of unknown parameters and the resulting minimax admissible procedures.

STAT 457 Statistical Learning II Units: 3.00
Introduction to the theory and application of statistical algorithms. Topics include classification, smoothing, model selection, optimization, sampling, supervised and unsupervised learning. Given jointly with STAT 857.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (STAT 362/3.0 and [ECON 351/3.0 or STAT 361/3.0]) or permission of the Department.
Offering Faculty: Faculty of Arts and Science

STAT 462 Statistical Learning I Units: 3.00
A working knowledge of the statistical software R is assumed. Classification; spline and smoothing spline; regularization, ridge regression, and Lasso; model selection; treed-based methods; resampling methods; importance sampling; Markov chain Monte Carlo; Metropolis-Hasting algorithm; Gibbs sampling; optimization. Given jointly with STAT 862.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite ([STAT 361 or ECON 351] and STAT 362) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Apply Markov Chain Monte Carlo for approximating the posterior distributions in Bayesian statistical Analysis.
2. Implement common algorithms in R for simulating random variables/vectors from standard and non-standard distributions.
3. Use standard Monte Carlo methods and importance sampling for approximating integrals, expectations and probabilities.
4. Understand common unsupervised learning methods including density estimation, clustering and dimension reduction techniques.
5. Understand the EM algorithm and its implementation in estimation for mixture models and censored data.
6. Understand the use of spline and penalization methods in supervised learning.

STAT 463 Fundamentals of Statistical Inference Units: 3.00
Decision theory and Bayesian inference; principles of optimal statistical procedures; maximum likelihood principle; large sample theory for maximum likelihood estimates; principles of hypotheses testing and the Neyman-Pearson theory; generalized likelihood ratio tests; the chi-square, t, F and other distributions.
Learning Hours: 132 (36 Lecture, 96 Private Study)
Requirements: Prerequisite STAT 269. Equivalency STAT 363. Recommended STAT 353.
Course Equivalencies: STAT363; STAT463
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Derive properties of distributions; finding optimal estimators and tests.
2. Develop a theoretical understanding of discrete and continuous random variables, distribution functions, sampling distributions, point estimation, interval estimation, hypothesis testing, large sample theory and basic Bayesian methods.
3. Prove Rao-Blackwell theorem, Lehmann-Scheffe theorem, Neyman-Pearson lemma.

STAT 464 Discrete Time Series Analysis Units: 3.00
Autocorrelation and autocovariance, stationarity; ARIMA models; model identification and forecasting; spectral analysis. Applications to biological, physical and economic data.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite STAT 361 or ECON 351 or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Deal with deterministic temporal structure in the data collected.
2. Use time-series models to do forecasting
3. Work with probabilistic analysis of regular time-series data.
4. Work with time-series data collected at regular intervals over time, e.g., daily temperature.

STAT 466 Statistical Programming with SAS and Applications Units: 3.00
Introduction to the basic knowledge in programming, data management, and exploratory data analysis using SAS software: data manipulation and management; output delivery system; advanced text file generation, statistical procedures and data analysis, macro language, structure query language, and SAS applications in clinical trial, administrative financial data.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite (STAT 263 or STAT 269) or permission of the Department.
Offering Faculty: Faculty of Arts and Science

STAT 471 Sampling and Experimental Design Units: 3.00
Simple random sampling; Unequal probability sampling; Stratified sampling; Cluster sampling; Multi-stage sampling; Analysis of variance and covariance; Block designs; Fractional factorial designs; Split-plot designs; Response surface methodology; Robust parameter designs for products and process improvement.
NOTE Offered jointly with STAT 871.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite ([STAT 361/3.0 or ECON 351/3.0] and STAT 463/3.0) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Develop a theoretical understanding of analysis of variance in simple comparative experiments, factorial designs, and fractional factorial designs with and without blocking; gain a deep understanding of optimal design theory.
2. Apply designs of experiments and sampling methods to collect data; analyze data from designs of experiments and different sampling schemes; draw conclusions from data analysis in various types of experiments.
3. Use functions in R packages and writing R codes to analyze data from designs of experiments and different sampling scheme.
4. Apply resampling methods such as bootstrap, Jackknife, balanced repeated replication for variance estimation.
5. Determine proper factorial experiments and sampling schemes for problems in applications.
6. Identify the problems of designs of experiments and/or survey sampling in many real applications.
7. Present topics of design of experiments and/or survey sampling clearly and effectively.

STAT 473 Generalized Linear Models Units: 3.00
An introduction to advanced regression methods for binary, categorical, and count data. Major topics include maximum-likelihood method, binomial and Poisson regression, contingency tables, log linear models, and random effect models. The generalized linear models will be discussed both in theory and in applications to real data from a variety of sources. Given jointly with STAT 873.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite ([STAT 361 or ECON 351] and STAT 463) or permission of the Department.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Compute numerical implementation of the scoring method for finding the maximum likelihood estimates in the cases of real and vector parameters.
2. Compute numerical solution of equations and numerical maximization of expressions depending on a real parameter.
3. Handle numerically various Poisson regression and binomial logistic regression models.
4. Identify the best unbiased estimates for the unknown parameters of exponential families of distribution.

STAT 486 Survival Analysis Units: 3.00
Introduces the theory and application of survival analysis: survival distributions and their applications, parametric and nonparametric methods, proportional hazards models, counting process and proportional hazards regression, planning and designing clinical trials. Given jointly with STAT 886.
Learning Hours: 120 (36 Lecture, 84 Private Study)
Requirements: Prerequisite ([STAT 361 or ECON 351] and STAT 463) or permission of the Department. Recommended STAT 462.
Offering Faculty: Faculty of Arts and Science
Course Learning Outcomes:

1. Analyze the real data set with R software using appropriate models.
2. Identify and classify data problems in survival analysis, define the appropriate survival function, distribution function, hazard function, and cumulative hazard function.
3. Understand and be able to compare survival functions of two or more populations.
4. Understand and be able to estimate survival functions using parametric, non-parametric, and semiparametric methods.

STAT 499 Topics in Statistics Units: 3.00
An important topic in statistics not covered in any other courses.
NOTE This course is repeatable for credit under different topic titles.
Requirements: Prerequisite Permission of the Department.
Offering Faculty: Faculty of Arts and Science

STAT 506 Topics in Statistics II Units: 3.00
An important topic in probability or statistics not covered in any other course.
Learning Hours: 132 (24 Individual Instruction, 108 Private Study)
Requirements: Prerequisite Permission of the Department.
Offering Faculty: Faculty of Arts and Science